SJHS
AP Calculus AB

Summer Assignment 2017

The attached assignment contains information you will need to know for AP Calculus AB. You are to read the information and work through the assigned problems during your summer break. You should have all work done and ready to turn in August 9, 2017. You will have a test on this material on Friday, August 11th. Do not wait until the week before school starts to begin this assignment. Work on it a little each day and before you know it you will be finished. If you need additional examples to help you with problems, khanacademy.com is a great resource. If you lose the packet, it's available on the SJHS website. I will be available at amanda.webb@stjosephhs.org if you have any questions on the assignment throughout the summer.

Remember:

1. **Do a little each day.**
2. **Show ALL work.**
3. **Be neat. Be very neat!!**
4. **Do all problems in the packet.**
   DON'T SIMPLIFY (unless instructed)
   DON'T RATIONALIZE (unless instructed)
5. **Have it all put together and ready to turn in August 9, 2017.**
6. **You can email me (amanda.webb@stjosephhs.org) at any time during the summer if you are having trouble!**

Have a great summer ☺

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$**. Find each.**

1. $f(2) =$ ____________
2. $g(-3) =$ ____________
3. $f(t+1) =$ ____________

4. $f\left[g(-2)\right] =$ ____________
5. $g\left[f(m+2)\right] =$ ____________
6. $\dfrac{f(x+h) - f(x)}{h} =$ ____________

Let $f(x) = \sin x$ **Find each exactly.**

7. $f\left(\dfrac{\pi}{2}\right) =$ ____________
8. $f\left(\dfrac{2\pi}{3}\right) =$ ____________

Let $f(x)=x^2,\ g(x)=2x+5,\ and\ \ h(x)=x^2-1$. **Find each.**

9. $h\left[f(-2)\right]=$ ________

10. $f\left[g(x-1)\right]=$ ________

11. $g\left[h(x^3)\right]=$ ________

Find $\dfrac{f(x+h)-f(x)}{h}$ **for the given function *f*.**

12. $f(x)=9x+3$

13. $f(x)=5-2x$

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y=x^2-2x-3$

x- int. (Let $y=0$)	y- int. (Let $x=0$)
$0=x^2-2x-3$	$y=0^2-2(0)-3$
$0=(x-3)(x+1)$	$y=-3$
$x=-1\ or\ x=3$	y- intercept $(0,-3)$
x- intercepts $(-1,0)\ and\ (3,0)$	

Find the x and y intercepts for each.

14. $y = 2x - 5$

15. $y = x^2 + x - 2$

16. $y = x\sqrt{16 - x^2}$

17. $y^2 = x^3 - 4x$

Use substitution or elimination method to solve the system of equations.
Example:

$$x^2 + y - 16x + 39 = 0$$
$$x^2 - y^2 - 9 = 0$$

Elimination Method

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x - 3)(x - 5) = 0$

$x = 3 \text{ and } x = 5$

Plug $x = 3$ *and* $x = 5$ into one original

$3^2 - y^2 - 9 = 0$ $\quad$ $5^2 - y^2 - 9 = 0$

$-y^2 = 0$ $\quad$ $16 = y^2$

$y = 0$ $\quad$ $y = \pm 4$

Points of Intersection $(5,4), (5,-4)$ *and* $(3,0)$

Substitution Method

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ (*The rest is the same as previous example*)

$x^2 - 8x + 15 = 0$

$(x - 3)(x - 5) = 0$

$x = 3 \text{ or } x - 5$

Find the point(s) of intersection of the graphs for the given equations.

18. $$\begin{aligned} x + y &= 8 \\ 4x - y &= 7 \end{aligned}$$

19. $$\begin{aligned} x^2 + y &= 6 \\ x + y &= 4 \end{aligned}$$

<u>Interval Notation</u>

20. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \le 4$		
	$[-1, 7)$	
		

Solve each equation. State your answer in BOTH interval notation and graphically.

21. $2x - 1 \ge 0$

22. $-4 \le 2x - 3 < 4$

23. $\dfrac{x}{2} - \dfrac{x}{3} > 5$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x) = \sqrt[3]{x+1}$	Rewrite f(x) as y
$y = \sqrt[3]{x+1}$	Switch x and y
$x = \sqrt[3]{y+1}$	Solve for your new y
$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3 = y+1$	Simplify
$y = x^3 - 1$	Solve for y
$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation

Find the inverse for each function.

24. $f(x) = 2x+1$

25. $f(x) = \dfrac{x^2}{3}$

Equation of a line

Slope intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Vertical line: x = c (slope is undefined)

Horizontal line: y = c (slope is 0)

26. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

27. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

28. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

29. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

30. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

31. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

32. Find the equation of a line passing through the points (-3, 6) and (1, 2).

33. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Trigonometric Equations:

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \le x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the end of the packet.)

34. $\sin x = -\frac{1}{2}$

35. $2\cos x = \sqrt{3}$

36. $\cos 2x = \frac{1}{\sqrt{2}}$

37. $\sin^2 x = \frac{1}{2}$

38. $\sin 2x = -\frac{\sqrt{3}}{2}$

39. $2\cos^2 x - 1 - \cos x = 0$

40. $4\cos^2 x - 3 = 0$

41. $\sin^2 x + \cos 2x - \cos x = 0$

<u>Limits</u>

Finding limits numerically.

Complete the table and use the result to estimate the limit.

42. $\lim_{x \to 4} \frac{x - 4}{x^2 - 3x - 4}$

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

43. $\lim_{x \to -5} \frac{\sqrt{4 - x} - 3}{x + 5}$

x	-5.1	-5.01	-5.001	-4.999	-4.99	-4.9
f(x)						

Finding limits graphically.

Find each limit graphically. Use your calculator to assist in graphing.

44. $\lim_{x \to 0} \cos x$

45. $\lim_{x \to 5} \frac{2}{x - 5}$

46. $\lim_{x \to 1} f(x)$

$$f(x) = \begin{cases} x^2 + 3, & x \neq 1 \\ 2, & x = 1 \end{cases}$$

<u>Evaluating Limits Analytically</u>

Solve by direct substitution whenever possible. If needed, rearrange the expression so that you can do direct substitution.

47. $\lim_{x \to 2} (4x^2 + 3)$

48. $\lim_{x \to 1} \frac{x^2 + x + 2}{x + 1}$

49. $\lim_{x \to 0} \sqrt{x^2 + 4}$

50. $\lim_{x \to p} \cos x$

51. $\lim_{x \to 1} \left(\frac{x^2 - 1}{x - 1} \right)$ HINT: Factor and simplify.

52. $\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}$

53. $\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$ HINT: Rationalize the numerator.

54. $\lim_{x \to 3} \frac{3 - x}{x^2 - 9}$

55. $\lim_{h \to 0} \frac{2(x+h) - 2x}{h}$

One-Sided Limits

Find the limit if it exists. First, try to solve for the overall limit. If an overall limit exists, then the one-sided limit will be the same as the overall limit. If not, use the graph and/or a table of values to evaluate one-sided limits.

56. $\lim_{x \to 5^+} \frac{x - 5}{x^2 - 25}$

57. $\lim_{x \to -3^-} \frac{x}{\sqrt{x^2 - 9}}$

58. $\lim_{x \to 10^+} \frac{|x - 10|}{x - 10}$

59. $\lim_{x \to 5^-} \left(-\frac{3}{x+5} \right)$

Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

60. $f(x) = \dfrac{1}{x^2}$

61. $f(x) = \dfrac{x^2}{x^2 - 4}$

62. $f(x) = \dfrac{2 + x}{x^2(1 - x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

63. $f(x) = \dfrac{x^2 - 2x + 1}{x^3 + x - 7}$

64. $f(x) = \dfrac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

65. $f(x) = \dfrac{4x^5}{x^2 - 7}$

Determine each limit as x goes to infinity.

RECALL: This is the same process you used to find Horizontal Asymptotes for a rational function.
**** In a nutshell**

1. Find the highest power of x.
2. How many of that type of x do you have in the numerator?
3. How many of that type of x do you have in the denominator?
4. That ratio is your limit!

66. $\lim\limits_{x\to\infty}\left(\dfrac{2x - 5 + 4x^2}{3 - 5x + x^2}\right)$

67. $\lim\limits_{x\to\infty}\left(\dfrac{2x - 5}{3 - 5x + 3x^2}\right)$

68. $\lim\limits_{x\to\infty}\left(\dfrac{7x + 6 - 2x^3}{3 + 14x + x^2}\right)$

Limits to Infinity

A rational function does not have a limit if it goes to $\pm\infty$, however, you can state the direction the limit is headed if both the left and right hand side go in the same direction.

Determine each limit if it exists. If the limit approaches ¥ *or* - ¥, please state which one the limit approaches.

69. $\lim\limits_{x \to -1^+} \dfrac{1}{x+1} =$ 70. $\lim\limits_{x \to 1^+} \dfrac{2+x}{1-x} =$ 71. $\lim\limits_{x \to 0} \dfrac{2}{\sin x} =$

72. Refer to the graph of $g(x)$ shown below in order to answer the following questions. If a limit does not exist explain why.

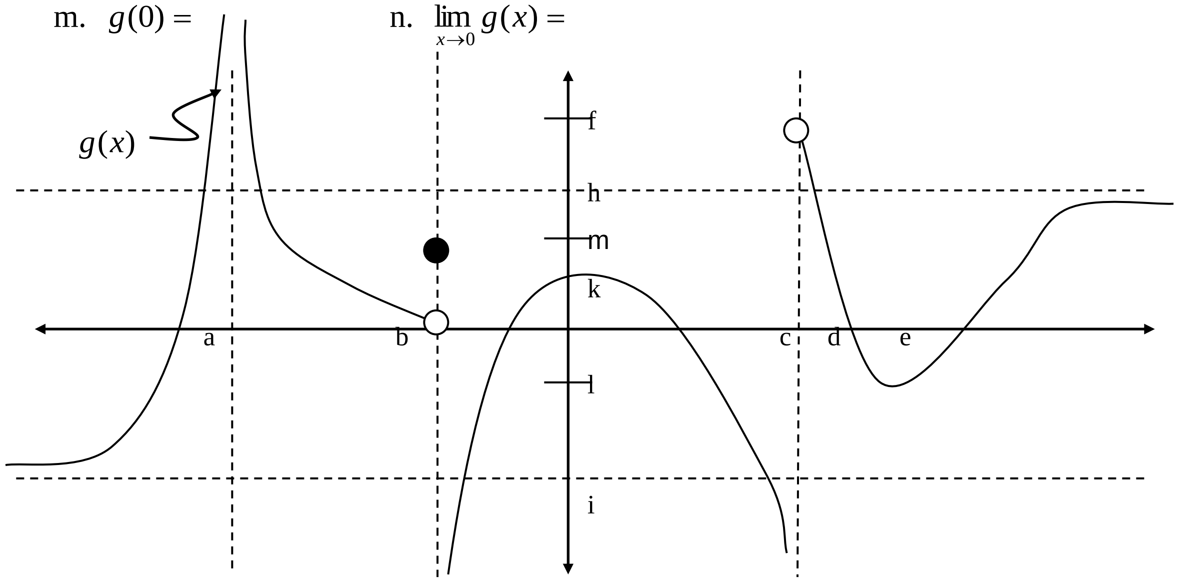
a. $\lim\limits_{x\to\infty} g(x) =$ b. $\lim\limits_{x\to-\infty} g(x) =$ c. $\lim\limits_{x\to a^+} g(x) =$

d. $\lim\limits_{x\to a^-} g(x) =$ e. $\lim\limits_{x\to a} g(x) =$ f. $\lim\limits_{x\to b^+} g(x) =$

g. $\lim\limits_{x\to b^-} g(x) =$ h. $\lim\limits_{x\to b} g(x) =$ i. $\lim\limits_{x\to c} g(x) =$

j. $\lim\limits_{x\to d} g(X) =$ k. $g(a) =$ l. $g(b) =$

m. $g(0) =$ n. $\lim\limits_{x\to 0} g(x) =$



Evaluate the following limits.

73. $\lim_{x \to 2} \frac{3x+4}{x+1}$

74. $\lim_{x \to 0^+} \sqrt{x}$

75. $\lim_{x \to 0^-} \sqrt{x}$

76. $\lim_{x \to 0} \sqrt{x}$

77. $\lim_{x \to 0} \frac{\sin x}{x}$

78. $\lim_{x \to \pi} \frac{\sin(x-\pi)}{x-\pi}$

79. $\lim_{h \to 0} \frac{2(3+h)-2(3)}{h}$

80. $\lim_{h \to 0} \frac{(x+h)^2-x^2}{h}$

Formula Sheet

Reciprocal Identities: $\csc x = \dfrac{1}{\sin x}$ $\sec x = \dfrac{1}{\cos x}$ $\cot x = \dfrac{1}{\tan x}$

Quotient Identities: $\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \dfrac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \dfrac{\log_b n}{\log_b a}$

Derivative of a Function: Slope of a tangent line to a curve or the derivative: $\lim\limits_{h \to \infty} \dfrac{f(x+h) - f(x)}{h}$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: Ax + By + C = 0